

## 4.9 Antiderivatives

NOT ON EXAM 3

antiderivative - reverse of derivative

→ given a function  $f(x)$  find  $F(x)$  such that  $F'(x) = f(x)$

for example,  $F(x) = -\cos x$  is the antiderivative of  $f(x) = \sin x$

because  $F'(x) = -(-\sin x) = \sin x = f(x)$

### Antiderivative of $x^n$

an antiderivative of  $x^n$  is  $\frac{x^{n+1}}{n+1}$  if  $n \neq -1$

why? because  $\frac{d}{dx} \left( \frac{x^{n+1}}{n+1} \right) = \frac{1}{n+1} (n+1) x^n = x^n$

for example, an antiderivative of  $x^2$  is  $\frac{x^{2+1}}{2+1} = \frac{x^3}{3} = \frac{1}{3} x^3$

check:  $\frac{d}{dx} \left( \frac{1}{3} x^3 \right) = \frac{1}{3} \cdot 3x^2 = x^2$

but notice  $\frac{1}{3}x^3 + 1$  is also an antiderivative of  $x^2$

because  $\frac{d}{dx} \left( \frac{1}{3}x^3 + 1 \right) = x^2$

in fact, so is  $\frac{1}{3}x^3 + 5$ ,  $\frac{1}{3}x^3 + \pi$ ,  $\frac{1}{3}x^3 + e^\pi - 10$ , etc

all are possible antiderivatives of  $x^2$

they differ only by a constant

to represent all of those, we write  $\frac{1}{3}x^3 + C$

some  
constant  
(constant of  
integration)

so, the most general antiderivative of  $x^n$  is

$$\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Example Find the antiderivative of  $f(x) = -\frac{10}{x^{12}}$

(what function has this as the derivative?)

rewrite:  $f(x) = \underbrace{-10}_{\text{constant multiple}} \underbrace{x^{-12}}_{x^n \text{ part}}$   
(doesn't participate just like in differentiation)

$$F(x) = -10 \left( \frac{x^{-12+1}}{-12+1} \right) + C$$

$$= -10 \left( \frac{x^{-11}}{-11} \right) + C = \boxed{\frac{10}{11} x^{-11} + C}$$

the indefinite integral is used to denote the process of finding the antiderivative

$$\int f(x) dx = F(x) + C$$

integral sign  
(stretched out "S")

part of the  
indefinite integral symbol

so, from last example, we know  $\int -\frac{10}{x^{12}} dx = \frac{10}{11} x^{-11} + C$

since finding antiderivative is closely related to differentiation, many rules and properties from derivative carry over.

for example, we can deal with terms of ~~Eq~~ separated by + or - individually

example

$$\int \left( \frac{2}{\sqrt{x}} + 2\sqrt{x} \right) dx$$

rewrite:  $\int (2x^{-1/2} + 2x^{1/2}) dx$

+ or - separate them  
deal with them one at a time

$$= 2 \left( \frac{x^{-1/2+1}}{-1/2+1} \right) + 2 \left( \frac{x^{1/2+1}}{1/2+1} \right) + C \quad \text{just need one } + C$$

$$= 2 \left( \frac{x^{1/2}}{1/2} \right) + 2 \left( \frac{x^{3/2}}{3/2} \right) + C$$

$$= \boxed{4x^{1/2} + \frac{4}{3}x^{3/2} + C}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{if } n \neq -1$$

what happens if  $n = -1$ ?

$$\int x^{-1} dx = \int \frac{1}{x} dx$$

what function has  
this as derivative?  $\rightarrow \ln x$

so,  $\int x^{-1} dx = \int \frac{1}{x} dx = \ln |x| + C$

need absolute value  
since  $x$  could be  
negative in  $\int \frac{1}{x} dx$

with trig functions, think of the differentiation rules in reverse

$$\int \cos x \, dx = \sin x + C \quad \text{because } \frac{d}{dx} (\sin x + C) = \cos x$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

and so on...

easy one:  $\int e^x dx = e^x + c$

example

$$\int \left( \sin x + \sec x \tan x + \frac{2}{x} - \frac{10}{x^{1/3}} \right) dx$$

separated by +/- so one at a time

$$= \int \left( \sin x + \sec x \tan x + 2x^{-1} - 10x^{-1/3} \right) dx$$

$$= -\cos x + \sec x + 2 \ln|x| - 10 \cdot \frac{x^{-1/3+1}}{-1/3+1} + C$$

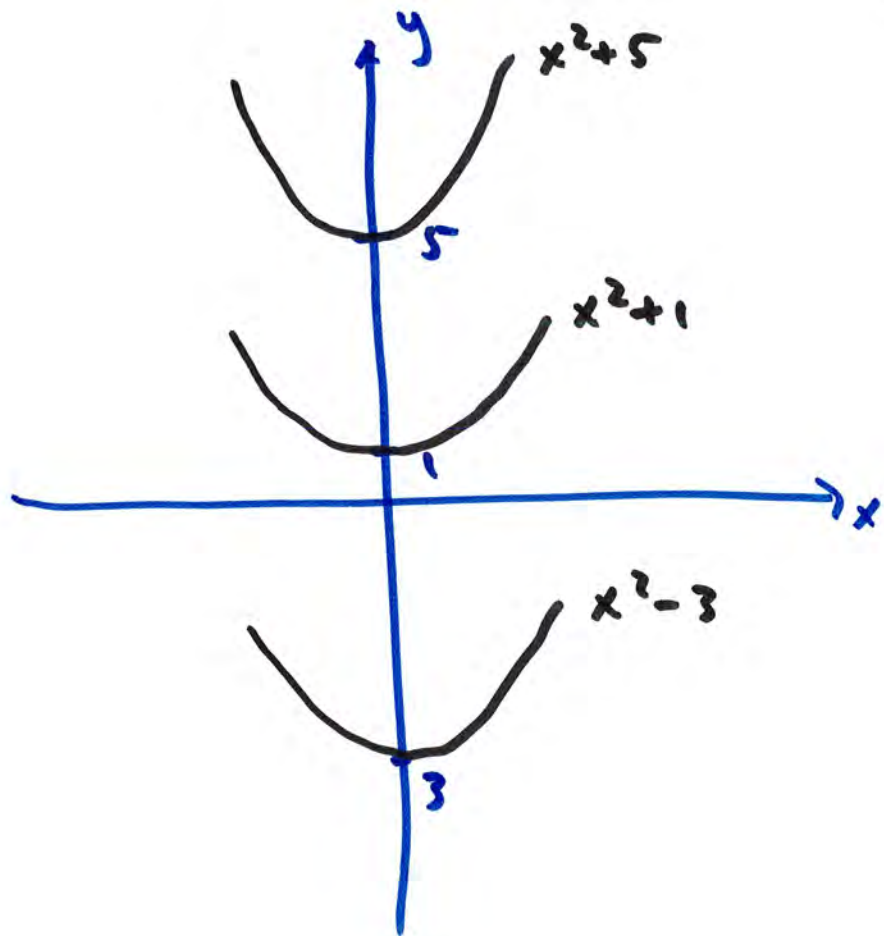
$$= -\cos x + \sec x + 2 \ln|x| - 10 \cdot \frac{x^{2/3}}{2/3} + C$$

$$= \boxed{-\cos x + \sec x + 2 \ln|x| - 15 x^{2/3} + C}$$

to find the value of "c" we need to know one point on the antiderivative

for example,  $\int 2x \, dx = x^2 + c$

parabola with vertex at  $(0, c)$



if we know one point that

$F(x) = x^2 + c$  passes through,

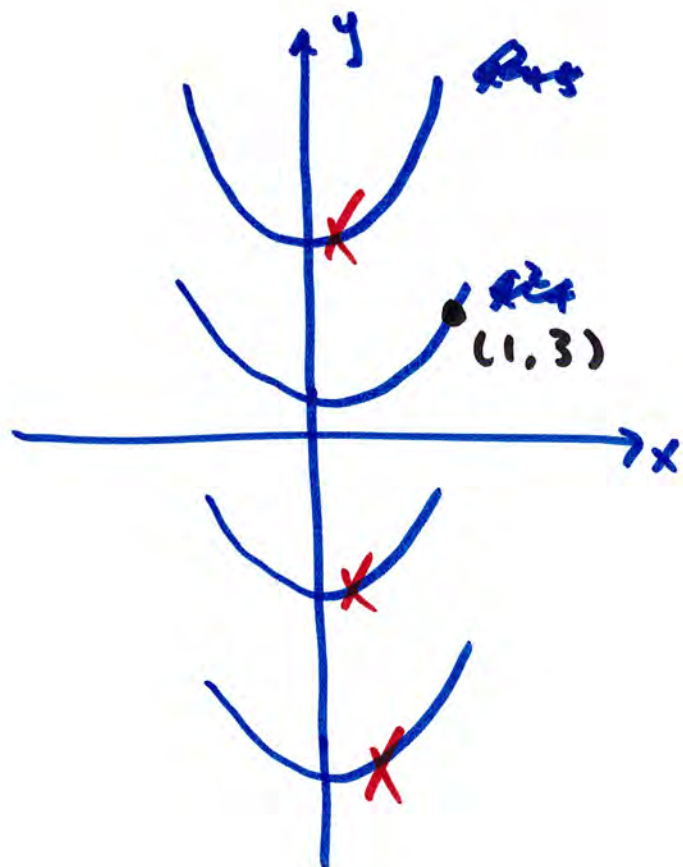
then we know which one it is

Example  $f(x) = 2x$

find  $F(x)$  such that  $F'(x) = f(x)$  and  $F(1) = 3$

$$F(x) = \int 2x \, dx = x^2 + C$$

$\underbrace{F(1) = 3}$   
 $(1, 3)$  is on  $F(x)$



$$F(x) = x^2 + C$$

Since  $(1, 3)$  is on  $F(x)$

Sub in  $x=1$ ,  $F(x) = 3$  to find  $C$

$$3 = 1^2 + C \quad C = 2$$

so,  $\boxed{F(x) = x^2 + 2}$

we handle higher-order derivatives the same way, but each order of derivative introduces one constant of integration

example  $F''(x) = \sin x$  find  $F(x)$

$$F'(x) = \int \sin x \, dx = -\cos x + C$$

$$F(x) = \int (-\cos x + C) \, dx$$

$$= -\sin x + Cx + D$$

second constant of integration

$$\boxed{F(x) = -\sin x + Cx + D}$$

to find C and D, we need one point on  $F(x)$  and one point on  $F'(x)$

for example, if we know (for last example)  $F'(0) = 3$ ,  $F(0) = 4$

from last page,  $F'(x) = -\cos x + C$

$(0, 3)$  on  $F'$  ←

plug in  $x=0$ ,  $F'=3$

$$3 = -\cos(0) + C$$

$$= -1 + C \rightarrow \boxed{C=4}$$

from last page,  $F(x) = -\sin x + Cx + D$   
(we now know  $C=4$ )

$$F(x) = -\sin x + 4x + D$$

$(0, 4)$  on  $F$  ←

plug in  $x=0$ ,  $F=4$

$$4 = -\sin(0) + 4(0) + D = D$$

$$\boxed{D=4}$$

so,  $\boxed{F(x) = -\sin x + 4x + 4}$

# MA 16100

## Exam 3

Tue., Apr. 12, 2022

6:30 p.m. – 7:30 p.m.

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